THE VISCOSITY OF A DISPERSION SYSTEM

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Hydrodynamic equations are presented for the viscous dispersion system on the basis of the interaction between the homogeneous medium and the nonuniformities. The equations have been confirmed via acoustic measurements.

There has long been interest in the equations of hydrodynamics to be applied to dispersion systems; in 1906, Einstein used hydrodynamic considerations to derive a relationship between the viscosity of a colloidal solution and the concentration of the dispersed phase [1]. However, up till now there have been no suitable formulas that take into account the viscosity as a function of the particle shape, size, and interaction. Here we give hydrodynamic equations for nonuniform media, which enable one to take into account these properties of the dispersed material. The equations have been tested by means of the acoustic dispersion in a two-phase system consisting of lycopodium plus an aqueous sodium chloride solution.

The velocity was measured by an interferometric method [2]; the error of measurement was 1.2%. The absorption was measured by a diffraction method [3]; the absorption coefficient was measured in this way to 15%. The speed of ultrasound and the absorption were measured over the range 4-30 mHz.

Lycopodium powder was used to prepare the mixture, the particles being nearly spherical in shape, and having a mean diameter of 30 μm. The particles were suspended in a solution of sodium chloride having a density equal to that of the particles ($\rho = 1.07$ g/cm^3).

Figure 1a shows the velocity as a function of frequency for a 9% concentration; there is no dispersion in the velocity within the error of the measurements. The absorption coefficient had a quadratic dependence on frequency only near 30 mHz. Figure 1b shows the absorption coefficient as a function of frequency for concentrations of 26, 17, 9%. The result is given as a ratio to the square of the frequency.

To make the Figure clear, the absorption values have been increased for $\varphi = 26\%$ by $2000 \cdot 10^{-17}$ sec^2/cm, the corresponding figure for $\varphi = 17\%$ being $1000 \cdot 10^{-17}$, and that for $\varphi = 9\%$ being reduced by $400 \cdot 10^{-17}$.

It is clear that there is a resonant effect; this follows directly from the theory of [4]. The broken line in Fig. 1b shows the calculation from Kasterin's formula for a resonator model. In the calculations, the natural frequency of the resonator was taken as 6.5 mHz, the pulsation amplitude of 10^{-8} cm, and the quantity characterizing the pulsation damping was taken as $0.25 \cdot 10^{-7}$ sec. The distance between the particles was calculated on the assumption that they form a cubic lattice. Kasterin considered that the displacement in the absorption band is determined by the distance between the particles. The above assumption about the disposition of the particles did not confirm the observed displacement for the band. The discrepancy is no doubt due to lack of accurate information on the regularity in particle disposition. To interpret the results we can use another approach from the equations of hydrodynamics. The main difficulty in writing the hydrodynamic equations for nonuniform medium is that the nonuniformity conflicts with the properties of a continuum. To avoid this conflict, we have to establish a rational method for replacing a substance with nonuniform physical properties by one with uniform ones. Predvoditelev has proposed a possible method for the purpose, and we use this here.

To write the equations of hydrodynamics we need to know how the tangential viscosity varies when there is nonuniformity; Einstein's method is very laborious and is not very clear as regards the assumptions and restrictions involved. Predvoditelev calculated the viscosity on the following basis. The nonuniform medium is represented as a system of hydrodynamic dipoles; the velocity potential for such dipoles takes the form

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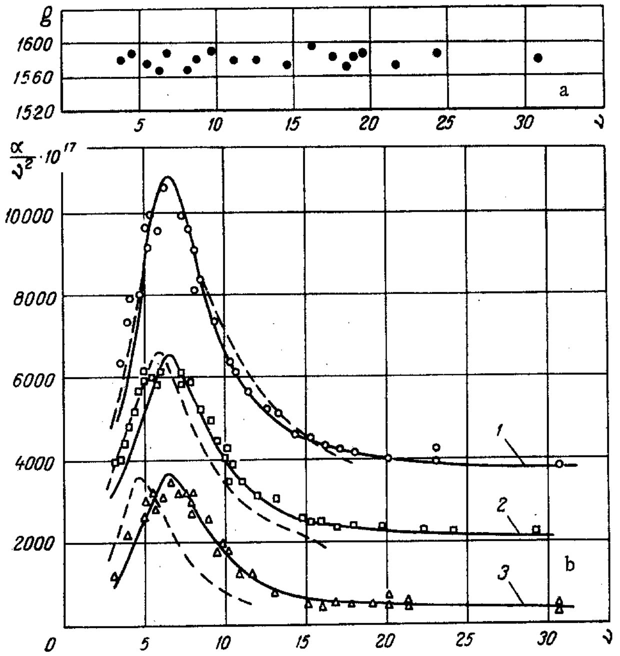

Fig. 1. a) Speed (m/sec) of ultrasound; b) absorption as functions of frequency for lycopodium of aqueous NaCl of strengths (%): 1) 26; 2) 17; 3) 9.

$$F = -\frac{Q}{R_1} + \frac{Q}{R_2}, \text{ where } R_1 = \sqrt{(x_1 - r)^2 + x_2^2 + x_3},$$

$$R_2 = \sqrt{(x_1 + r)^2 + x_2^2 + x_3},$$

where 2r is the length of a dipole and Q is source strength. Knowing the potential, we can calculate the velocity W_0 = grad F at any point in the medium and therefore such quantities as

$$W_0 = \sum_{i=1}^{3} \left(\frac{\partial F}{\partial x_i} \right)^2 \text{ and } \delta_0^2 = \sum_{i=1}^{3} \left(\frac{\partial W_{0i}}{\partial x_i} \right)^2.$$

These quantities are necessary to calculate the energy that is transformed into heat in the homogeneous liquid. The energy loss due to the Stokes resistance is equal to the products of the frontal resistance by the velocity W_0 for the flow of the uniform liquid, $E_S = 9\eta_0 W_0^2 \varphi / r^2$; here φ is particle concentration and η_0 is the viscosity of the uniform medium. The following is the energy loss due to deformation of the velocity vector:

$$E_\delta = 2\eta_0 \delta_0^2.$$

The total energy loss can then be put as

$$E = E_s + E_\delta = 2\eta_0 \delta_0^2 \left(1 + \frac{\varphi}{2r^2 S} \right).$$

In the latter equation we have used $W_0^2 = 9/2 \cdot \delta_0^2/S$, and S is as follows for this velocity potential:

$$S = \frac{1}{R_1 R_2} \cdot \frac{q \frac{R_1^3 R_2^3}{3} + \frac{R_2^3}{R_1^3} \sum \alpha_{i1}^4 + \frac{2}{3} q (R_1^3 - R_2^3) \sum \alpha_{i1}^2 \alpha_{i2}^2}{\frac{R_2^2}{2R_1^2} + \frac{R_1^2}{2R_2^2} - \sum \alpha_{i1} \alpha_{i2}},$$

where $q = R_2^3 - R_1^3 / R_1^3 R_2^3$, while α_{i1} and α_{i2} are the direction cosines of vectors $\mathbf{R}_1$ and $\mathbf{R}_2$. As W_0 is related to the actual velocity of the nonuniform medium W by

$$(1 - \varphi)\, \mathbf{W}_0 = \mathbf{W},$$

we have

$$E = 2\eta_0 \frac{1+\dfrac{\varphi}{2r^2S}}{(1-\varphi)^2}\delta^2.$$

From the latter relationship we find the viscosity of the nonuniform medium as

$$\eta(\varphi) = \eta_0 \frac{1+\dfrac{\varphi}{2r^2S}}{(1-\varphi)^2}. \tag{1}$$

For spherical particles, $R_1 = R_2 = r$, $\alpha_{11} = \alpha_{12} = \sqrt{2}/2$, $\alpha_{21} = \alpha_{22} = -\sqrt{2}/2$, $r^2S = 1$ and (1) becomes Einstein's formula [1].

The viscosity enables us to use the ordinary rule to write the stress tensor and the equation of motion for the nonuniform liquid:

$$\bar{\rho}\frac{\partial \mathbf{W}}{\partial t} = -\operatorname{grad}\bar{p} + \eta(\varphi)\Delta\mathbf{W} + \frac{\eta(\varphi)}{3}\operatorname{grad}\operatorname{div}\mathbf{W}. \tag{2}$$

Here $\bar{\rho}$ and $\bar{p}$ are respectively the density and pressure in the nonuniform medium.

The caloric equation of state may be put in general form as

$$\bar{p} = f(\bar{\rho}). \tag{3}$$

The equation of continuity is written as in filtration theory [5]. If the number of particles is unaltered, this equation takes the form

$$(1-\varphi)\frac{\partial \rho_0}{\partial t} + \operatorname{div}\rho_0\mathbf{W} = 0, \tag{4}$$

where ρ_0 is the density of the uniform medium. Equations (2)-(4) are sufficient to solve the acoustic problem. We use the approach of [6] to get the absorption coefficient as

$$\alpha = \frac{2\eta(\varphi)\omega^2}{3\bar{g}^3\bar{\rho}}, \tag{5}$$

where $\bar{g} = \sqrt{d\bar{p}/d\bar{\rho}}$, $\omega = 2\pi\nu$ is the frequency of the ultrasound.

To bring the measured results into accordance with (5), we have to make an assumption about the interaction of the wave front with the nonuniformities, which can be solved by the theory of manifold [7].

Consider pendulum-type motion at any point in an elastic and liquid substance; if from that point there proceeds out a perturbation in the form of a surface $\xi(x, y, z, t) = \text{const}$, then there must be a velocity of this perturbation $g_\xi = -\partial\xi/\partial t \cdot 1/H_\xi$. Here H_ξ is the first differential parameter of the function ξ, $H_\xi = d\xi/dn$. We have $g_\xi = -i\omega\xi/H_\xi$ for a perturbation of periodic character, and the equation for the pendulum takes the form

$$-\omega_0^2 + 2h\omega i = \frac{g_\xi^2 H_\xi^2}{\xi^2}, \tag{6}$$

where ω_0 is the natural frequency of the pendulum oscillation and h is the damping coefficient.

We assume further that in the medium there is another surface $\psi(x, y, z, t) = C$, which satisfies the wave equation with a periodic solution, so $\omega = \omega_1 + i\delta$, and then the wave equation can be put in the form

$$g_\psi^2\Delta\psi + (\omega_1 + i\delta)^2\psi = 0. \tag{7}$$

The state of the manifold is characterized by the two surfaces ξ and ψ. The relationship is given by Green's formula

$$\int \frac{d\psi}{dn}\cdot\frac{d\xi}{dn}d^3x + \int \xi\Delta\psi d^3x = \int \xi\frac{d\psi}{dn}dS.$$

If we assume $g_\xi = g_\psi = \bar{g}$, then (6) and (7) go with

$$\int \xi^2\frac{d\psi}{dn}\left(\frac{d\xi}{dn}\right)^{-1}d^3x = -\int \xi\psi d^3x \tag{8}$$

to give us Green's formula

$$\int \frac{\xi\psi}{\bar{g}^2}\, d^3x = \frac{\int \xi \frac{d\psi}{dn}\, dS}{(\omega_0^2 - \omega_1^2 + \delta^2) - 2(h+\delta)\,\omega_1 i}\,. \tag{9}$$

It is readily seen that (8) applies if the gradient of the product of $\xi\psi$ is zero.

We calculate the integrals of (9) for a manifold representing a medium filled with particles. We assume that the medium carries an acoustic wave, whose potential is $\psi' = \psi + \psi_0$, where ψ_0 is the velocity potential in the uniform medium and ψ is the potential arising from the nonuniformities. We put $\xi = \bar{\rho}$, $W = d\psi/dn$; the integral over the surface is transformed into a volume integral, and we use the following relationships:

$$\frac{\delta p}{\rho_0} = -\frac{\partial\psi}{\partial t}, \quad \frac{\partial\psi}{\partial t} = i\omega_1\psi, \quad \frac{\partial\rho}{\partial t} = i\omega_1\delta\rho.$$

Then formula (9) becomes

$$n^2 = 1 + \frac{\omega_1^2}{(\omega_0^2 - \omega_1^2 + \delta^2) + 2(h+\delta)\,\omega_1 i}\,,$$

where $n^2 = g_0^2/\bar{g}^2$ is the complex refractive index. The absorption coefficient α of the nonuniform medium is

$$\alpha = \frac{(h+\delta)\,\omega_1^4}{\bar{g}n^2\,[(\omega_0^2 - \omega_1^2 + \delta^2)^2 + 4(h+\delta)^2\,\omega_1^2}\,. \tag{10}$$

The latter formula has been derived from purely kinematic consideration. We can compare (10) with (5), which is derived from the equations of hydrodynamics, which enables us to determine the mode of flow around the nonuniformities. If we put $\delta = 0$, $2\eta_0\varphi/[3\bar{\rho}\bar{g}^3(1-\varphi)^2] = h/\bar{g}n^2f^2$, then with the form factor r^2S we get

$$\frac{1}{2r^2S} = \frac{1}{2r^2S_0} + \frac{\nu^2f^2}{(\nu^2 - \nu_0^2)^2 + 4h^2\nu^2}\,.$$

Here r^2S_0 is the form character characterizing the flow around the nonuniformities outside the resonance region.

The solid line in Fig. 1b has been constructed from (5). The form factor was calculated from (11). The calculations were performed for

$$\nu_0 = 6.5\cdot 10^6\ \frac{1}{\text{sec}}\,,\ \frac{1}{2r^2S_0} = 170,\quad h = 2.56\cdot 10^6\ \frac{1}{\text{sec}}\ ,\ f = 1.4\cdot 10^8\ \frac{1}{\text{sec}}\,.$$

The absorption curves for the other concentrations were calculated on the assumption that the form factor increases linearly with the concentration. Figure 1 shows that (5) decribes satisfactorily the frequency and concentration relationships for the absorption coefficient.

From these results one concludes that one can use Predvoditelev's generalized hydrodynamic equations to describe nonuniform medium. In these equations the viscosity of the nonuniform system is determined via Newton's differential equation. The viscosity formula contains the form factor, which may differ between instruments for measuring viscosity, which shows that one can determine the viscosity by means of Newton's equation without resort to Bingham's equation [8].

LITERATURE CITED

1. A. Einstein, Ann. d. Physik., No. 4, 324, 301 (1906).
2. G. Willard, J. Acoust. Soc. Amer., 23, No. 1, 83 (1951).
3. V. V. Vladimirskii and M. D. Galanin, Zh. Éksp. Teor. Fiz., 9, 233 (1939).
4. N. P. Kasterin, Wave Propagation in an Inhomogeneous Medium [in Russian], Moscow (1903).
5. N. E. Zhukovskii, Zh. Russ. Fiz.-Khim. Obshch., Fizika, 21, 1 (1889).
6. P. Appel', Mechanics [in Russian], Moscow (1911), vol. 3.
7. A. S. Predvoditelev, in; Heat and Mass Transfer [in Russian], Énergiya, Moscow (1970), p. 151.
8. E. Bingham, Fluidity and Plasticity, New York, London (1922).